

Harmonic Structure of Tracheal Biometrics

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ABSTRACT

According to existing research, in 20% of patients the required tracheal size (diameter and length) could not be measured automatically (using software solution). In this paper, we use Zederbauer's harmonic circle to define the relationship between a person's body height and the diameter and length of their trachea. We then explore differences by gender. The results show that there is a highly significant correlation between measured data and values obtained by harmonic analysis. The final goal is to obtain the mathematical interpretation of the relationship of the tracheal size to the height of man, in order to further apply such sizes in the creation of 3D models that could finally be printed on appropriate 3D printers and with the suitable material.

Key words: anthropometry, harmonic circle, proportions of the human body

Introduction

Changes in tracheal dimensions occur in a variety of conditions, and the knowledge of normal tracheal dimensions is essential to the diagnosis of these conditions.¹ Trachea is a cartilaginous-membranous tube that with its branches ensures the supply of air to the lungs. It is made of 16–20 cartilages in the shape of a horseshoe (ellipse) and fibroelastic membrane.²

Trachea, as a direct continuation of the larynx goes from the lower edge of the annular cartilage (cricoid cartilage), i.e. the sixth neck (cervical) to the fourth thoracic vertebra and where it is divided into two main bronchi. The place where the trachea divides into bronchi is called bifurcatie trachea – midpoint trachea (tracheal bifurcation) or carina, and it is projected on the front wall of the chest at the level of the angulus steric (sternal angle). Rear projection of the midpoint of the trachea fits the pointed tip of the adapter (tip of the spinous process) of the third thoracic vertebrae or the line joining the two trigonum spines of the scapula (interscapular line). Observing at the transversal cut, the trachea is not completely round. The anterior and lateral walls of the trachea are rounded and the back is straight. The back wall is called paries membranaceus.³

There are two parts to the trachea: the neck and the chest. The neck of the trachea is located in the medial line, and the thorax lies slightly more on the right side. Therefore, a midpoint of the trachea is located a little to the right of the midline. The shape of the trachea is in the form of a roller which is slightly flattened at the back. The tracheal lumen is not equal in length. Just below the larynx, the trachea is slightly narrowed (thyroid impression). Then the lumen expands to the middle of the trachea, only to narrow again, because the aortic impression arises on the left side. The healthy trachea lumen is always open and wider than the larynx lumen.³

The function of the airway is to conduct air to the alveoli, where gas from the inhaled air and gasses dissolved in the blood of the alveolar capillaries are exchanged. The rings of the cartilage trachea are associated with fibroelastic and muscle tissue. Theoretically, such a structure exists to prevent loss of tracheal stability. Tracheal stability means maintaining the shape of the trachea not to bend or recede, but rather to keep its lumen and shape the same when moving the neck and head.

According to literature, the average adult trachea measures about 11 cm in length with a diameter that varies from 2 to 2.5 cm.⁴ However, previously published figures

for normal tracheal dimensions show considerable variation in attempts to establish normal ranges for these measurements.¹ It has been shown that tracheal dimensions vary by age, gender and population.^{5,6} Measurements on chest radiographs have shown that the coronal diameter of the normal trachea ranges from 13 to 25 mm in men and from 10 to 21 mm in women, and the sagittal diameter of the normal trachea ranges from 13 to 27 mm in men and from 10 to 23 mm in women.⁷

Some studies have reported the association between tracheal width and body height which makes it possible to predict the tracheal dimensions from simple external measurements.^{6,8} However, according to the study by Ma et al.⁹, in 20% of patients the required tracheal size (diameter and length) could not be measured automatically (using software solution) from other measurements.

In a previous paper, we have shown that the method of calculation in the canon of the harmonic circle is advantageous in defining sizes required by anthropometric analyses or those that we have no ability of measuring, as it leads to a high degree of correlation with actual measurements on a representative sample.¹⁰

In this paper, we use the harmonic circle to define the relationship between a person's body height and the diameter and length of their trachea. We then explore differences by gender. The final goal is to obtain the mathematical interpretation of the relationship of the tracheal size to the height of man, in order to further apply such sizes in the creation of 3D models that could finally be printed on appropriate 3D printers and with the suitable material.

Material and Method

In this research we observed 79 people, whose head, neck and lungs were recorded using computer tomography (CT).¹¹ In our sample, the organs were normally developed, without any deformation of the trachea and bronchi. The type of CT device on which we recorded the subjects was MSCT Simens Somatom definition AS. We divided patients by gender into 42 males and 37 females for measurement of height and weight. We selected candidates between 40 and 50 years of age. The anthropometric measurements were taken by a protocol described in detail elsewhere.¹²

From CT recordings tracheal length (TL), anteroposterior (APD) – sagittal cut and laterolateral (LLD) – frontal cut of the trachea were measured in males and females. We further calculated the ratio of tracheal length to body height for each patient separately. From this ratio, we calculated the mean that we used as a benchmark. Furthermore, we derived a value from Zederbauer harmonic circle, that is a linear combination of the sizes shown and calculated on the circle.^{13,14} We compared the calculated values with the measured methods using statistical methods. Finally, we determined whether such a coefficient resulted in the true trachea size for a given height of a person.

Harmonic circle

A significant contribution to the definition of the length of body parts is given by the Zederbauer harmonic circle,^{13,14} which is shown in Figure 1. The construction of the harmonic circle is based according to Zederbauer¹³ on a regular triangle with hypotenuse a , and sides b , as it is shown in Figure 1. After we draw over the sides of the triangle the respective squares, we can finally draw the circle of radius R that passes through the peaks of a such geometrical construction. Mathematical relations between these geometrical values give harmonic numbers. In the Zederbauer's circle the standing height of man is used as diameter, this height being equal to the sum of eight head lengths, and after that, in a geometrical combination, other characteristic body dimensions are inserted. As previously described, the derived lengths of different body segments already showed strong correlation with human anthropometric measures.^{10, 15}

The mathematical relations in the harmonic circle are described in Table 1.

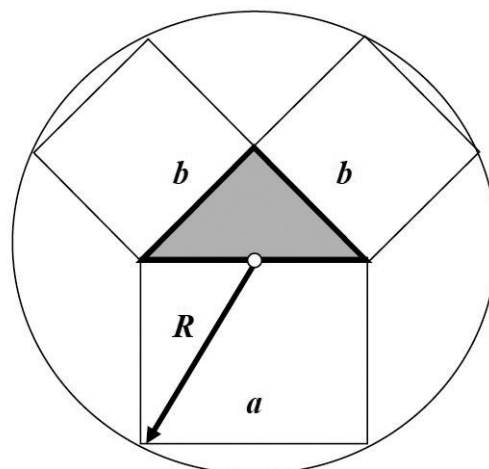


Fig. 1. Harmonic circle.

Results

Length measurements and diameters of trachea

Table 2 presents the obtained measures (mean, SD and range) in mm for males. The mean values for BH, TL, APD and LLD were 1736.15 mm, 132.69 mm, 17.56 mm and 18.04 mm, respectively. The ranges of the examined variables were 1650–1890 (BH), 120–150 (TL), 16–19 (APD) and 17–21 (LLD).

If we return to the harmonic circle of the mean value of the standing height, we will associate it with the variable H that in the harmonic circle represents the diameter $H = 2R$. For the so defined variable R we get the derived values of the harmonic variables in the circle shown in Table 3. The calculated harmonic sizes are BH – 1736.15 mm, TL – 137.24 mm, APD – 17.36 mm and LLD – 19.41 mm.

TABLE 1
HARMONIC SIZES AS HARMONIC NUMBERS

Basic harmonic sizes			Derived harmonic sizes		
Label	Formula	Calculated values	Label	Formula	Calculated values
a	1a	1	r	$= \frac{\sqrt{2}-1}{2} = b - \frac{a}{2}$	0.207
b	$b = \frac{\sqrt{2}}{2}$	0.707	d	$= \frac{\sqrt{5}-1}{2} = R - \frac{a}{2}$	0.618
R	$R = \frac{\sqrt{5}}{2}$	1.118	b + r	$= \frac{\sqrt{2}-1}{2}$	0.914

a = Hypotenuse of the harmonic triangle; b = Cathetus of the harmonic triangle side; R = Radius of the harmonic circle. Source: Muftić et al.¹⁵ and Mičković et al.¹⁰

Table 4 presents slightly lower obtained measures (mean, SD and range) in mm for females. The mean values for BH, TL, APD and LLD were 1649.28 mm, 127.57 mm, 17.32 mm and 17.35mm, respectively. The ranges of the examined variables were 1550–1780 (BH), 105–148 (TL), 15–20 (APD) and 15–19.8 (LLD).

By using the same calculation as for males, we obtained the derived harmonic values for females shown in Table 5. The calculated harmonic sizes are BH –1649.28 mm, TL – 130.38 mm, APD – 16.49 mm and LLD – 18.43 mm.

A correlation analysis with two sets of measured and calculated values was performed. The results are presented in Table 6. It can be seen that the Pearson correlation coefficient is 0.999, which indicates an extremely high degree of correlation. Further rank correlation analyses (Kendall's tau_b and Spearman's rho), given in Table 7, confirmed a highly significant correlation between the measured and the calculated harmonic values.

Volume of trachea

The cross-section of the trachea is the ellipse of the known tracheal size (diameter AP and diameter LL), and therefore we were able to calculate the surface section. The cross-section of the trachea is approximately an ellipse and in the present work we abstracted its imperfections caused by different structures and thicknesses of the tissue at the front and rear part of the trachea.¹⁶ The area of the section is given by the formula presented below (although it is generally known) to show how one can calculate the surface section and from that to calculate the

volume showing that mathematized access to medical problems exist as a realistic option.

The surface of ellipse is as follows:

$$\begin{aligned}
 P &= 4 \int_0^a y dx = 4 \int_0^a b \cdot \sqrt{1 - \frac{x^2}{a^2}} dx = [x = a \sin t, dx = a \cos t dt] \\
 &= 4 \int_0^{\frac{\pi}{2}} b \cdot \sqrt{\cos^2 t} \cdot \cos t dt = 4ab \int_0^{\frac{\pi}{2}} \cos^2 t dt \\
 &= 4ab \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2t}{2} dt = 4ab \int_0^{\frac{\pi}{2}} \frac{dt}{2} + ab \int_0^{\frac{\pi}{2}} \cos 2t dt \\
 &= 4ab \cdot \frac{\pi}{2} \Big|_0^{\frac{\pi}{2}} + ab \sin 2t \Big|_0^{\frac{\pi}{2}} = ab\pi.
 \end{aligned}$$

From this formula, the surface of the trachea can be calculated as:

$$P = ab\pi \tag{Formula 1}$$

Then, let a = Diameter AP (PAP) and let b = Diameter LL (PLL) define the ellipse as the base of the cylinder of a given surface according to the Formula 1, the height of which is in reality the length of the trachea so that the cross-sectional area of the trachea will be:

$$P = PAP * PLL * \pi \tag{Formula 2}$$

And the cylinder with the elliptic base

$$V = ab\pi * h \tag{Formula 3}$$

will for trachea be:

$$Vm = PAP * PLL * DD * \pi \tag{Formula 4}$$

TABLE 2
MALES - MEASURED SIZES IN MM

A _n	A _n (mean)	SD	Var	Min	Max
Body height BH	1736.15	68.01207	4625.641	1650	1890
Trachea length TL	132.69	10.53079	110.8974	120	150
Trachea diameter APD	17.56	0.970725	0.942308	16	19
Trachea diameter LLD	18.04	0.964033	0.929359	17	21

TABLE 3
MALES - CALCULATED SIZES IN MM

A_n	Measured values A_n	Harmonic sizes	Harmonic sizes (H)	Calculated harmonic values A_n
BH	1736.15	2R	H	1736.15
TL	132.69	R*b	0.07905378H	137.24
APD	17.56	2R/100	0.01 H	17.36
LLD	18.04	b	0.01118 H	19.41

TABLE 4
FEMALES - MEASURED SIZES IN MM

A_n	A_n (mean)	SD	Var	Min	Max
Body height BH	1649.28	66.38632	4407.143	1550	1780
Trachea length TL	127.57	11.17493	124.8791	105	148
Trachea diameter APD	17.32	1.301732	1.694505	15	20
Trachea diameter LLD	17.35	1.242671	1.544231	15	19.8

TABLE 5
FEMALES - CALCULATED SIZES IN MM

A_n	Measured values A_n	Harmonic sizes	Harmonic sizes (H)	Calculated harmonic values A_n
BH	1649.28	2R	H	1649.28
TL	127.57	R*b	0.07905378H	130.38
APD	17.32	2R/100	0.01 H	16.49
LLD	17.35	b	0.01118 H	18.43

TABLE 6
PEARSON CORRELATION

		Mean values of measured sizes	Calculated harmonic sizes
Mean values of measured sizes	Pearson Correlation	1	.999**
	Sig. (2-tailed)		.000
	N	25	25
Calculated harmonic sizes	Pearson Correlation	.999**	1
	Sig. (2-tailed)	.000	
	N	25	25

On the other hand, we have calculated the values of AP Diameter and LL Diameter in the canon of the Zederbauer circle, which are functionally related to the height of the BH, so it follows that

$$PAP = a_1 * H \quad PLL = a_2 * H \quad DD = a_3 * H \quad \text{Formula 5}$$

And if we include these calculated values in Formula 4, we obtain the following:

$$Vr = a_1 a_2 a_3 * H^3 * \pi \quad \text{Formula 6}$$

The results of the tracheal volume calculation are expressed in cm^3 and presented in Table 8.

We can see that the difference between the calculated and the measured results amounts to 3% of volume for the female data, and 10% for the male data where there is a slightly larger deviance in the calculated length of the trachea. For both males and females the calculated values give larger volume with acceptable variation.

Conclusion

In this paper, we presented a new approach of the analysis of anthropometric dimensions of the parts of the human body that are not available for direct mea-

TABLE 7
RANK CORRELATION COEFFICIENTS

Tests		Calculated harmonic sizes	
Kendall's tau_b	Mean values of measured sizes	Correlation Coefficient	.990**
		Sig. (2-tailed)	.000
		N	25
	Correlation Coefficient	.973**	
		Sig. (1-tailed)	.000
		N	18
Spearman's rho	Mean values of measured sizes	Correlation Coefficient	.999**
		Sig. (2-tailed)	.000
		N	25
	Correlation Coefficient	.995**	
		Sig. (1-tailed)	.000
		N	18

** Correlation is significant at the 0.01 level.

TABLE 8
TRACHEAL VOLUME IN CM³

	Volume from measured quantities Vm	Volume from calculated quantities Vr	Relative difference
Male	132.05	145.27	10.02%
Female	120.43	124.48	3.36%

surement in all situations. It represents an advancement and better feasibility in determining sizes of body parts in relation to similar studies⁹ in the percentage of matched cases between the measured and calculated values. The problems of deformation of body parts and their replacement by 3D printed models require a mathematical approach to the estimation of their sizes. The trachea has proven to be one such component that can

(in the future) be replaced partly or as a whole, for which measurement problems may occur in various diseases or injuries. This paper demonstrates an approach to the estimation and calculation of anthropometric trachea sizes showing a high degree of correlation with the actual measured values on a representative sample. It can be helpful in forensics, biomedical engineering, ergonomics and surgery.

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HARMONIJSKA STRUKTURA BIOMETRIJE DUŠNIKA

SAŽETAK

Istraživanja pokazuju da kod 20 % pacijenata nije moguće automatski softverskim programom izmjeriti potrebne veličine dušnika (promjer i duljina). U ovom radu pomoću Zederbauerove harmonijske kružnice definiramo odnos tjelesne visine čovjeka i promjera i duljine njegovog dušnika. Potom razlučujemo taj odnos prema spolu. Odredili smo i volumen dušnika kao važnu veličinu u matematičkom modelu. Konačni cilj i put kojim treba krenuti je matematičko definiranje odnosa dušnika prema visini čovjeka u cilju daljnje primjene takvih veličina u kreiranju 3D modela koji bi se konačno mogli isprintati na odgovarajućim printerima i u odgovarajućem materijalu, za potrebe biomehaničkog inženjersva, kirurgije i forenzike.